

An approach to non-leptonic B-decays on the lattice

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New Horizons for Lattice Computations with Chiral Fermions

with

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Outline

Nonleptonic B decays (CKM)
Weak operators
Four-point functions on the lattice

Starting points:
Chiral Perturbation Theory
Resonance contribution
Hard pion ChPT

$B^- \rightarrow D^0 P^-$ ← experimentally accessible

$B^- \rightarrow \bar{D}^0 P^-$

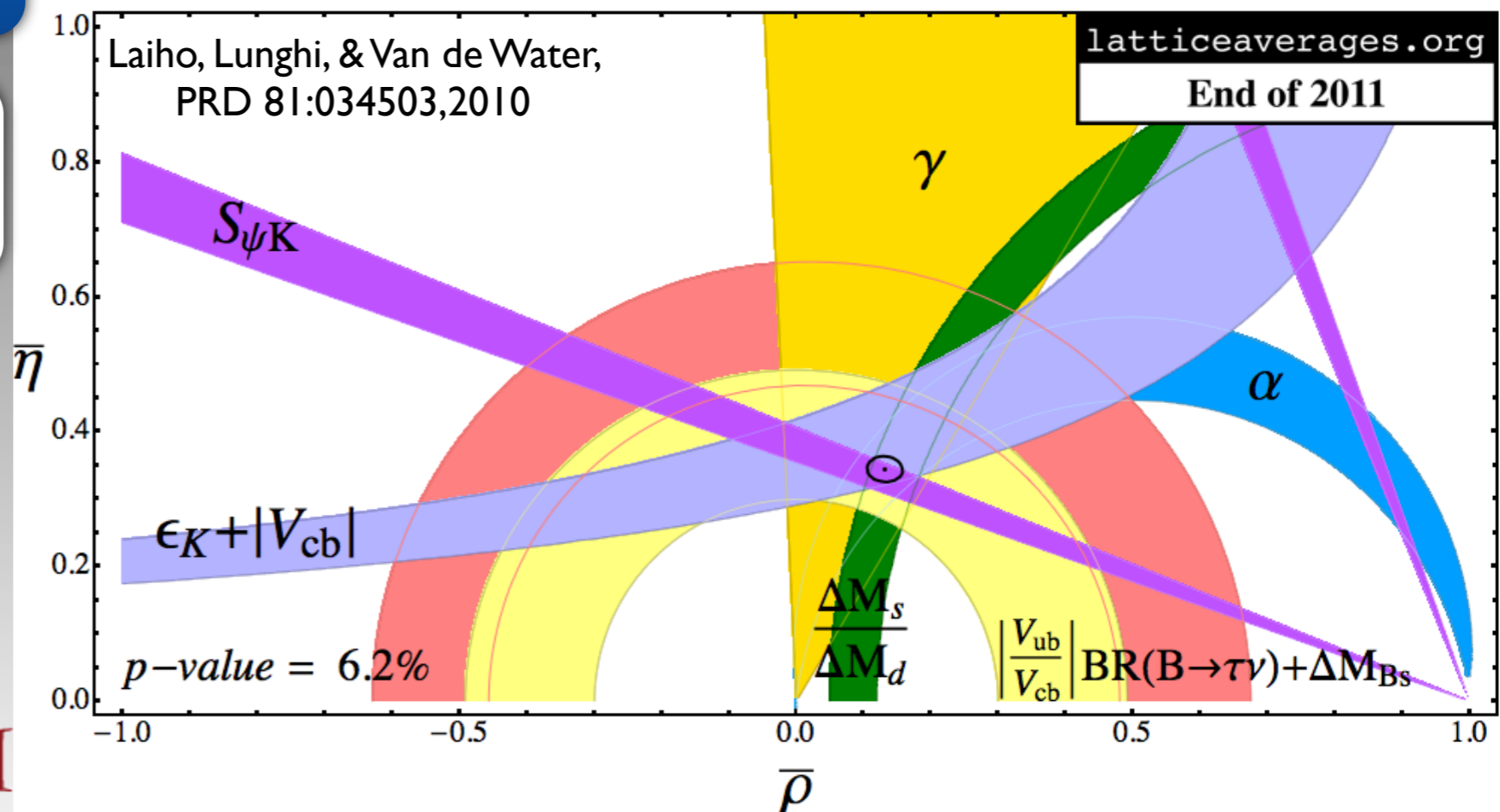
The ratio

$$r_{BP} = \frac{\text{Br}[B^- \rightarrow \bar{D}^0 P^-]}{\text{Br}[B^- \rightarrow D^0 P^-]}$$

can give insight into γ
(HFAG, arXiv:1010.1589)

On the lattice, define
a reduced ratio:

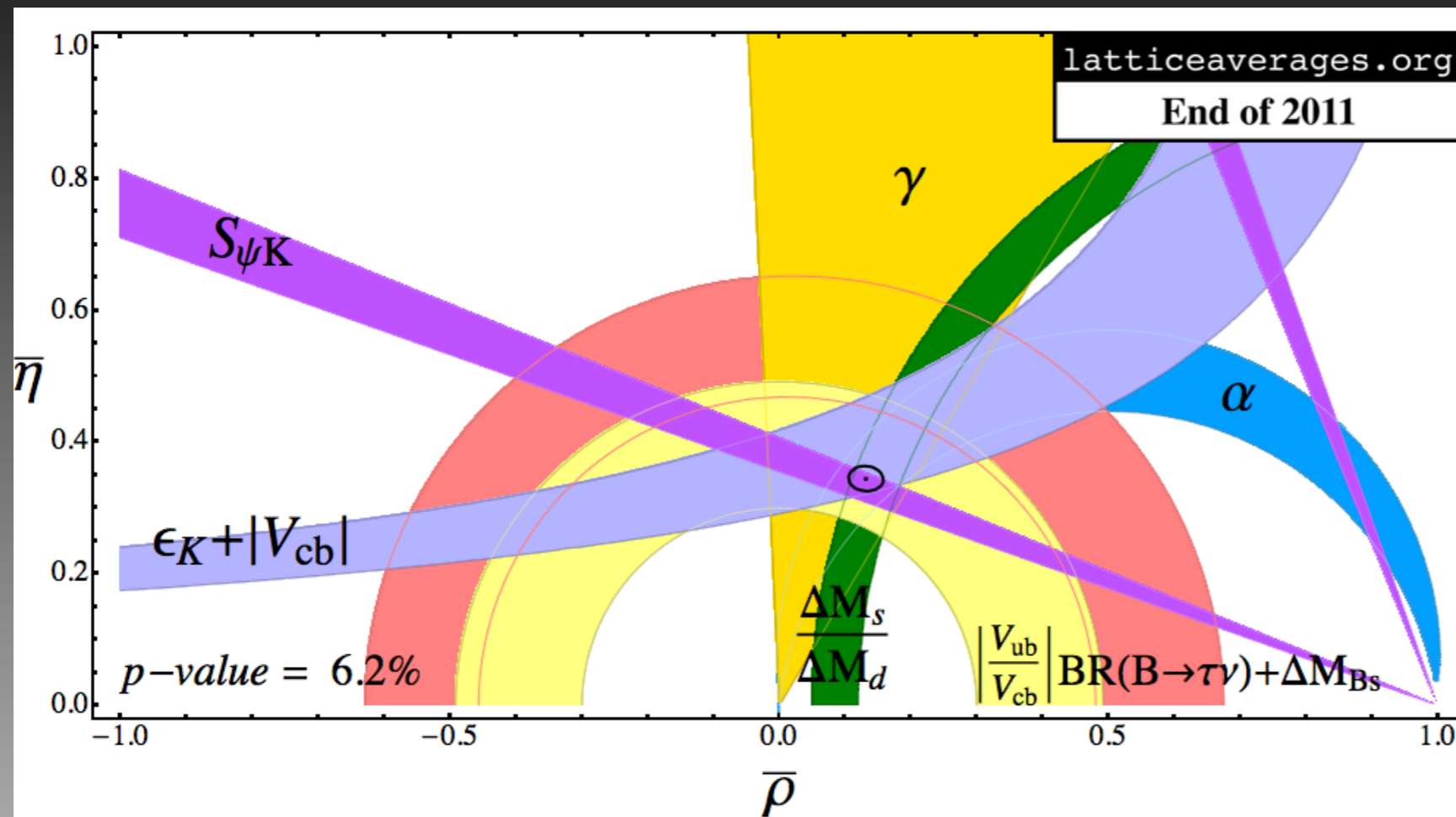
$$r_{BP}^{\text{red}} \equiv \frac{r_{BP}}{V_{CKM}^{\text{combo}}} = r_{BP} \frac{|V_{cb}^* V_{uq}|^2}{|V_{ub}^* V_{cq}|^2}$$



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$$B^- \rightarrow D^0 P^- \quad B^- \rightarrow \bar{D}^0 P^-$$



$$\gamma \quad O(25\%)$$

$$\alpha \quad O(5\%)$$

$$\beta \quad O(3\%)$$

Hope:

Determination of the real part
of these amplitudes to 15-20% [~ 5 years?]

$$\Rightarrow \gamma \sim 10\%$$



H
T

Weak operators

$$Q_1^{b \rightarrow c, i} = (\bar{q}_\alpha^i \gamma^\mu (1 - \gamma_5) b_\alpha) (\bar{c}_\beta \gamma_\mu (1 - \gamma_5) u_\beta)$$

$$Q_2^{b \rightarrow c, i} = (\bar{q}_\alpha^i \gamma^\mu (1 - \gamma_5) b_\beta) (\bar{c}_\beta \gamma_\mu (1 - \gamma_5) u_\alpha)$$

$$Q_1^{b \rightarrow \bar{c}, i} = (\bar{q}_\alpha^i \gamma^\mu (1 - \gamma_5) b_\alpha) (\bar{u}_\beta \gamma_\mu (1 - \gamma_5) c_\beta)$$

$$Q_2^{b \rightarrow \bar{c}, i} = (\bar{q}_\alpha^i \gamma^\mu (1 - \gamma_5) b_\beta) (\bar{u}_\beta \gamma_\mu (1 - \gamma_5) c_\alpha)$$

Maiani-Testa theorem

$$\langle 0 | \pi D \mathcal{O}_{\text{weak}} B | 0 \rangle$$

4pt functions only yields real part (no strong phase)

Can be circumvented
(Lellouch-Lüscher, RBC/UKQCD)

$$\text{e.g., } K \rightarrow 2\pi$$



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Heavy-light meson ChPT

$$\mathcal{L}_G = \frac{f^2}{8} \text{Tr} (\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + \frac{1}{4} \mu f^2 \text{Tr} (\mathcal{M} \Sigma + \mathcal{M} \Sigma^\dagger)$$

$$\Sigma \longrightarrow L \Sigma R^\dagger, \text{ where } L \in \text{SU}(3)_L, \text{ and } R \in \text{SU}(3)_R$$

$$H_{v,a}^{(\bar{Q})} = \left(\gamma^\mu \mathcal{V}_{\mu,a}^{*(\bar{Q})} - \gamma_5 \mathcal{P}_a^{(\bar{Q})} \right) \left(\frac{1 - \not{v}}{2} \right) \leftarrow \text{HL fields with anti-quark}$$

$$H_{v,a}^{(Q)} = \left(\frac{1 + \not{v}}{2} \right) \left(\gamma^\mu \mathcal{V}_{\mu,a}^{*(Q)} - \gamma_5 \mathcal{P}_a^{(Q)} \right) \leftarrow \text{HL fields with quark}$$

$$\mathbb{V}_\mu = \frac{i}{2} [\sigma^\dagger \partial_\mu \sigma + \sigma \partial_\mu \sigma^\dagger]$$

$$\mathbb{A}_\mu = \frac{i}{2} [\sigma^\dagger \partial_\mu \sigma - \sigma \partial_\mu \sigma^\dagger]$$

$$H_Q(x) \rightarrow S H_Q(x) \mathbb{U}^\dagger(x), \quad \bar{H}_Q(x) \rightarrow \mathbb{U}(x) \bar{H}_Q(x) S^\dagger$$

$$H_{\bar{Q}}(x) \rightarrow \mathbb{U}(x) H_{\bar{Q}}(x) S^\dagger, \quad \bar{H}_{\bar{Q}}(x) \rightarrow S \bar{H}_{\bar{Q}}(x) \mathbb{U}^\dagger(x)$$

$$S \in U(4)$$

$$\sigma = \sqrt{\Sigma} = e^{i\Phi/f}$$

$$\sigma(x) \rightarrow L \sigma(x) \mathbb{U}^\dagger(x) = \mathbb{U}(x) \sigma(x) R^\dagger$$

$$\mathcal{L}_{\text{HL},1} = -i \text{Tr}(\bar{H} H v \cdot \overleftrightarrow{D}) + g_\pi \text{Tr}(\bar{H} H \gamma^\mu \gamma_5 \mathbb{A}_\mu)$$



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Heavy-light meson ChPT

Chiral-level Weak operators:

$b \rightarrow c$ operators

$$\mathcal{O}_{\chi,i} = \sum_x \left\{ \alpha_{1,x} \text{Tr}_D \left[\left(\sigma_{1k} \overline{H}_{v',k}^{(c)} \right) \Gamma_2 \Xi'_x \Xi_x \Gamma_1 \left(H_{v,l}^{(b)} \sigma_{li}^\dagger \right) \right] + \alpha_{2,x} \text{Tr}_D \left[\left(\sigma_{1k} \overline{H}_{v',k}^{(c)} \right) \Gamma_2 \Xi'_x \right] \text{Tr}_D \left[\Xi_x \Gamma_1 \left(H_{v,l}^{(b)} \sigma_{li}^\dagger \right) \right] \right\}$$

$b \rightarrow \bar{c}$ operators

$$\overline{\mathcal{O}}_{\chi,i} = \sum_x \left\{ \overline{\alpha}_{1,x} \text{Tr}_D \left[\Xi'_x \Gamma_2 \left(\overline{H}_{v',k}^{(\bar{c})} \sigma_{k1}^\dagger \right) \Xi_x \Gamma_1 \left(H_{v,l}^{(b)} \sigma_{li}^\dagger \right) \right] + \overline{\alpha}_{2,x} \text{Tr}_D \left[\Xi'_x \Gamma_2 \left(\overline{H}_{v',k}^{(\bar{c})} \sigma_{k1}^\dagger \right) \right] \text{Tr}_D \left[\Xi_x \Gamma_1 \left(H_{v,l}^{(b)} \sigma_{li}^\dagger \right) \right] \right\}$$

$$\{\Xi'_x, \Xi_x\} = \left\{ \{1, 1\}, \{\gamma_\nu, \gamma^\mu\}, \{\not{v}', \not{v}\}, \{\not{v}', 1\}, \{1, \not{v}\}, \{\sigma_{\mu\nu}, \sigma^{\mu\nu}\}, \right. \\ \left. \{\gamma_5, \gamma_5\}, \{\gamma_\mu \gamma_5, \gamma^\mu \gamma_5\}, \{\not{v}' \gamma_5, \not{v} \gamma_5\}, \{\not{v}' \gamma_5, \gamma_5\}, \{\gamma_5, \not{v} \gamma_5\} \right\}$$



Heavy-light meson ChPT

$$\begin{aligned}\mathcal{O}_{\chi,i} = & [\beta_1 + (\beta_1 + \beta_2) (v' \cdot v)] \left[\left(\sigma_{1k} \mathcal{P}_k^{(c)\dagger} \right) \left(\mathcal{P}_l^{(b)} \sigma_{li}^\dagger \right) \right] \\ & + [(\beta_1 - \beta_2) v'^\mu - \beta_1 v^\mu] \left[\left(\sigma_{1k} \mathcal{P}_k^{(c)\dagger} \right) \left(\mathcal{V}_{\mu,l}^{*(b)} \sigma_{li}^\dagger \right) \right] \\ & + [\beta_1 v'^\mu - (\beta_1 + \beta_2) v^\mu] \left[\left(\sigma_{1k} \mathcal{V}_{\mu,k}^{*(c)\dagger} \right) \left(\mathcal{P}_l^{(b)} \sigma_{li}^\dagger \right) \right] \\ & - 4 [(\beta_1 - \beta_2) + \beta_1 (v' \cdot v)] \left[\left(\sigma_{1k} \mathcal{V}_{\mu,k}^{*(c)\dagger} \right) \left(\mathcal{V}_l^{*(b)\mu} \sigma_{li}^\dagger \right) \right],\end{aligned}$$

$$\begin{aligned}\overline{\mathcal{O}}_{\chi,i} = & [\overline{\beta}_1 + \overline{\beta}_2 (v' \cdot v)] \left[\left(\mathcal{P}_k^{(\bar{c})\dagger} \sigma_{k1}^\dagger \right) \left(\mathcal{P}_l^{(b)} \sigma_{li}^\dagger \right) \right] \\ & - [\overline{\beta}_2 v'^\mu - (\overline{\beta}_1 + \overline{\beta}_5) v^\mu - \overline{\beta}_3 (v' \cdot v) v^\mu] \left[\left(\mathcal{P}_k^{(\bar{c})\dagger} \sigma_{k1}^\dagger \right) \left(\mathcal{V}_{\mu,l}^{*(b)} \sigma_{li}^\dagger \right) \right] \\ & + [\overline{\beta}_1 v'^\mu - \overline{\beta}_2 v^\mu] \left[\left(\mathcal{V}_{\mu,k}^{*(\bar{c})\dagger} \sigma_{k1}^\dagger \right) \left(\mathcal{P}_l^{(b)} \sigma_{li}^\dagger \right) \right] \\ & + [4\overline{\beta}_2 - \overline{\beta}_3 - 2 (\overline{\beta}_1 + \overline{\beta}_4 + \overline{\beta}_5) (v' \cdot v)] \left[\left(\mathcal{V}_{\mu,k}^{*(\bar{c})\dagger} \sigma_{k1}^\dagger \right) \left(\mathcal{V}_l^{*(b)\mu} \sigma_{li}^\dagger \right) \right],\end{aligned}$$



Leading order



$$\langle D^0 K^- | \mathcal{O}_{\chi,s} | B^- \rangle = \langle D^0 \pi^- | \mathcal{O}_{\chi,d} | B^- \rangle = \frac{i}{f} \langle D^- | \mathcal{O}_{\chi,s} | B^- \rangle,$$

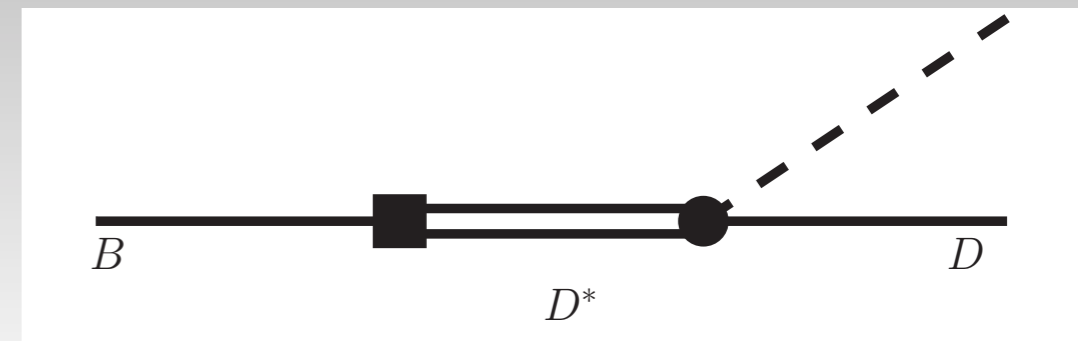
$$\langle \bar{D}^0 K^- | \bar{\mathcal{O}}_{\chi,s} | B^- \rangle = \langle \bar{D}^0 \pi^- | \bar{\mathcal{O}}_{\chi,d} | B^- \rangle = \frac{i}{f} \langle D^- | \bar{\mathcal{O}}_{\chi,s} | B^- \rangle .$$

similar expressions for $K \rightarrow 2\pi$
[Bernard et al, PRD32 2343]

Can also include resonances:



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Of course here the physical point has

$$p_\pi = p_D \approx 2 \text{ GeV}$$

Not appropriate for chiral expansion unless
one takes the unphysical point

$$m_B \approx m_D$$

But then we're far from the point of interest.

How to go beyond tree-level?



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Beyond tree-level:

Hard-pion ChPT (HPChPT)

Exploit the fact that hard scales
can be absorbed into LEC's

Flynn & Sachrajda, Nucl.Phys. B812, 64 $K_{\ell 3}$

Bijnens & Celis, Phys.Lett. B680, 466 $K \rightarrow 2\pi$

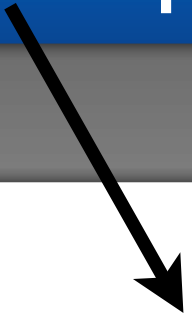
Bijnens & Jemos, Nucl.Phys. B840, 54 $B \rightarrow \pi$



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In the SU(2) chiral theory,
we have the generic one-loop form

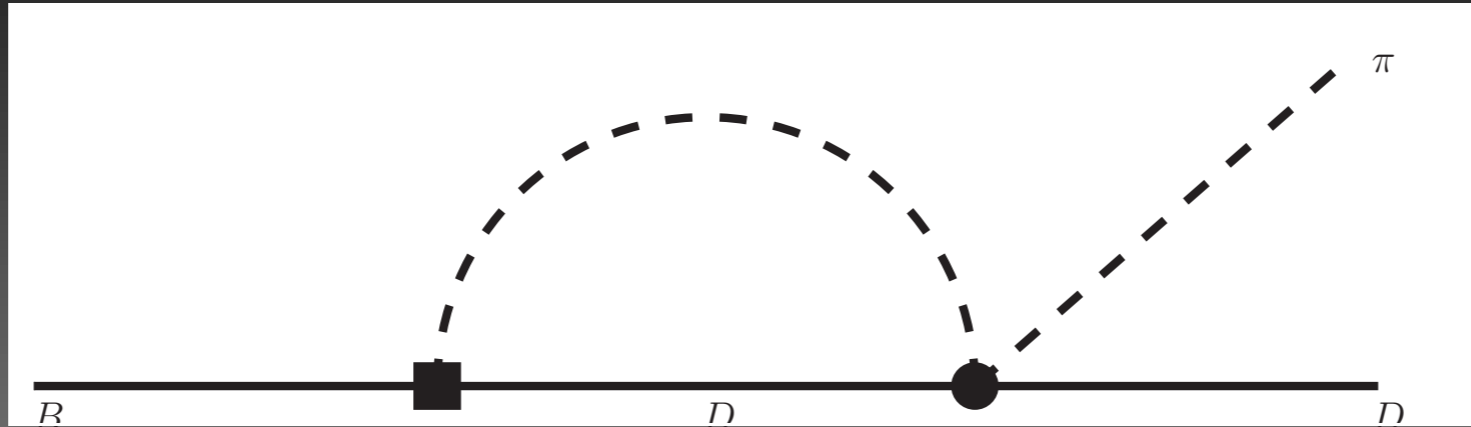
tree-level amplitude


$$\mathcal{M} = \mathcal{M}^{\text{tree}} \left[1 + a \frac{m_\pi^2}{16\pi^2 f^2} \ln \left(\frac{m_\pi^2}{\Lambda^2} \right) + L m_\pi^2 \right]$$

a and *L* are LEC's that depend on the hard scales



HPChPT example



$$\frac{\langle D^0 \pi^- | \mathcal{O}_{\chi,d} | B^- \rangle^{\text{tree}}}{8f^2} \int \frac{d^d \ell}{(2\pi)^d} \frac{i}{\ell^2 - m_\pi^2 + i\epsilon} \frac{iv' \cdot (\ell - p_\pi)}{v' \cdot (\ell - k - p_\pi) - \Delta + i\epsilon},$$

$$\frac{\langle D^0 \pi^- | \mathcal{O}_{\chi,d} | B^- \rangle^{\text{tree}}}{8} I,$$

$$I = \frac{1}{16\pi^2 f^2} \left[\frac{v' \cdot k + \Delta}{v' \cdot (k + p_\pi) + \Delta + i\epsilon} I_2(m_\pi, v' \cdot (k + p_\pi) + \Delta + i\epsilon) - m_\pi^2 \ln \left(\frac{m_\pi^2}{\Lambda^2} \right) \right]$$



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$$I = \frac{1}{16\pi^2 f^2} \left[\frac{v' \cdot k + \Delta}{v' \cdot (k + p_\pi) + \Delta + i\epsilon} I_2(m_\pi, v' \cdot (k + p_\pi) + \Delta + i\epsilon) - m_\pi^2 \ln \left(\frac{m_\pi^2}{\Lambda^2} \right) \right]$$

in the limit $v' \cdot k \gg m_\pi$

$$I_2(m_\pi, v' \cdot (k + p_\pi) + \Delta) \approx -m_\pi^2 \ln \left(\frac{m_\pi^2}{\Lambda^2} \right)$$

Inject momentum into weak vertex so that...



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$$p_\pi \approx 0$$

$$I(p_\pi \approx 0) \rightarrow -2 \frac{m_\pi^2}{16\pi^2 f^2} \ln \left(\frac{m_\pi^2}{\Lambda^2} \right)$$

$$p_\pi \approx k$$

$$I(p_\pi \approx k) \rightarrow -\frac{3}{2} \frac{m_\pi^2}{16\pi^2 f^2} \ln \left(\frac{m_\pi^2}{\Lambda^2} \right)$$

Different contributions to a, L in above expression

Both a and L have unknown dependence on kinematics, but pion mass dependence is known.



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Conclusion

Problem is far from solved

Need full one-loop calculation

Need lattice calculations (takers?)

Key: Feasible problem for lattice to tackle



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